Credit Shock and a Two-sector Model Extension

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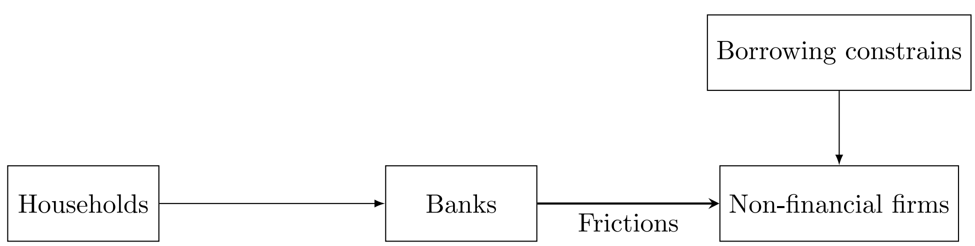
Abstract

This paper focuses on the macroeconomic implications of credit-supply-side financial shocks, particularly how disruptions in banks' balance sheets propagate through the broader economy. While much of the literature has emphasized credit-demand shocks, this study shifts the focus to the supply side, where financial intermediaries face constraints that limit their lending capacity. Extending the Gertler and Karadi (2011) framework, the chapter introduces a DSGE model with two sectors, aiming to capture core mechanisms behind the Great Recession. A key contribution is the analysis of how shocks originating in one sector, such as the housing sector, can transmit to other sectors through the banking system, as banks reallocate or reduce credit in response to changing risk and regulatory conditions. The model highlights how leverage heterogeneity and intersectoral linkages via financial intermediaries can amplify the macroeconomic effects of financial disruptions, offering insights into the sectoral dynamics observed during the Great Recession.

1 Introduction

In the period leading up to the Great Recession, credit was particularly abundant in the housing sector, resulting in significantly higher leverage relative to other sectors. The deterioration of the housing sector eroded banks' net worth, thereby constraining the credit they could extend to other sectors. Motivated by this observation, I extend the Gertler-Karadi model to a two-sector framework. This allows for the analysis of how shocks originating in a highly leveraged sector, such as housing sector, transmit to a less leveraged sector, and how disruptions in financial intermediation affect the broader economy.

Figure 1: Financial frictions affecting credit demand



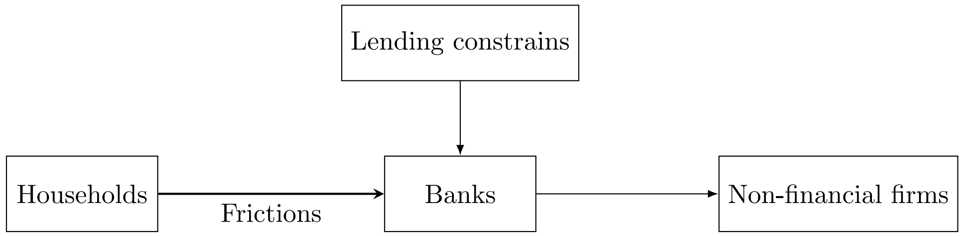
The extended model retains the central mechanism of financial frictions, i.e. intermediaries face constraints in raising external funds (instead of the non-financial firms). As such, the focus remains on how financial shocks propagate through the intermediary sector, consistent with the mechanisms that were prominent during the Great Recession.

The following sections describe the extended model, emphasizing the new assumptions and structural features. After calibrating the model, I perform quantitative experiments to investigate the model's dynamic responses and policy implications.

2 Two-sector DSGE Model

In this section, I will introduce the two-sector model extended from Gertler and Karadi (2011). This is a monetary DSGE model with nominal rigidities developed by CEE and SW. Financial frictions exist

Figure 2: Financial frictions affecting credit supply



in the process of transferring funds from households to financial intermediaries, where in the event that financial intermediaries divert funds from the project, there is a restriction on the ability of financial intermediaries to lend funds to non-financial firms. Explicitly characterized financial intermediaries are included in the model in order to study the channel of shock propagation from the credit-supply side.

For tractability, I abstract from unconventional credit policies and instead focus on a setting with two sectors, each producing a distinct type of capital good. These capital goods are used as inputs by intermediate goods producers. Retail firms operate under monopolistic competition and use intermediate goods to produce final output. The central bank is assumed to follow a conventional monetary policy rule, adjusting the nominal interest rate as its primary instrument.

In particular, I will study the capital quality shock, which is a simple way of introducing an exogenous source of variation in the value of capital. A negative capital quality shock in one sector generates deterioration of banks' balance sheet, especially their net worth, and affects their access to and cost of funding and their ability to make new loans to the other sector. The market price of capital will be endogenous and the capital quality shock will serve as an exogenous trigger for the asset price dynamics.

I also examine two types of financial shocks that influence credit supply: one that directly affects banks' leverage ratios, and another that captures changes in financial regulation, reflecting the extent of risk control imposed on the banking system.

2.1 Households

There is a continuum of identical households of measure unity, consisting of bankers and workers, with the fractions being constant over time, but an individual can switch between the two occupations.

The household preference is given by

$$\max E_t \sum_{i=0}^{\infty} \beta^i [\ln(C_{t+i} - hC_{t+i-1}) - \frac{\chi_t}{1+\varphi} L_{t+i}^{1+\varphi}] \tag{1}$$

where C_t is consumption, L_t is family labor supply and χ_t is the labor supply shock. This shock may serve as a proxy for various labor market frictions that are relevant in real-world settings. $0 < \beta < 1$ is the subjective discount factor. $0 < h < 1$ is the habit persistence parameter, and $\varphi > 0$.

The household budget constraint is given by

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1} \tag{2}$$

where B_{t+1} is the total quantity of short term debt the household acquires, W_t is the real wage, Π_t is the net payouts to the household from ownership of both non-financial and financial firms, and T_t is the lump sum taxes.

Then the household's first order conditions for labor supply and consumption/saving are the following:

$$\rho_t W_t = \chi_t L_t^{\varphi} \tag{3}$$

with

$$\rho_t = (C_t - hC_{t-1})^{-1} - \beta h E_t (C_{t+1} - hC_t)^{-1} \tag{4}$$

and

$$E_t \beta \Lambda_{t,t+1} R_{t+1} = 1 \tag{5}$$

with

$$\Lambda_{t,t+1} \equiv \frac{\rho_{t+1}}{\rho_t} \tag{6}$$

2.2 Financial intermediaries

Financial intermediaries lend funds obtained from households to non-financial firms from two sectors. The balance sheet of a banker/intermediary j at the end of period t is

$$Q_{1t}S_{1jt} + Q_{2t}S_{2jt} = Q_tS_t = N_{jt} + B_{jt+1} \tag{7}$$

where Q_{it} is the relative price of each claim for sector i, S_{ijt} is the quantity of financial claims on non-financial firms in sector i. B_{jt+1} is the deposits the intermediary obtains from households (debt), and N_t is the net worth amount that a banker/intermediary j has at the end of period t (equity capital). The intermediary earns from the assets invested in sector i with stochastic return R_{ikt+1} over the period t to $t+1$, and pays the household the non-contingent real gross return R_{t+1}. Both R_{ikt+1} and R_{t+1} will be determined endogenously.

Define

$$\kappa_{jt} = \frac{Q_{1t}S_{1jt}}{Q_tS_{jt}} \tag{8}$$

as the share of banks' asset in sector 1. Therefore, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$\begin{aligned} N_{jt+1} &= R_{1kt+1}Q_{1t}S_{1jt} + R_{2kt+1}Q_{2t}S_{2jt} - R_{t+1}B_{jt+1} \\ &= (R_{1kt+1} - R_{t+1})Q_{1t}S_{1jt} + (R_{2kt+1} - R_{t+1})Q_{2t}S_{2jt} + R_{t+1}N_{jt} \\ &= [(R_{1kt+1} - R_{t+1})\kappa_{jt} + (R_{2kt+1} - R_{t+1})(1-\kappa_{jt})]Q_tS_{jt} + R_{t+1}N_{jt} \end{aligned} \tag{9}$$

The banker's objective is to maximize expected terminal wealth, given by

$$\begin{aligned} V_{jt} &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i\beta^i\Lambda_{t,t+1+i}(N_{jt+1+i}) \\ &= \max E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i\beta^{i+1}\Lambda_{t,t+1+i}[(R_{1kt+1} - R_{t+1})Q_{1t}S_{1jt} + (R_{2kt+1} - R_{t+1})Q_{2t}S_{2jt} \\ &\quad + R_{t+1}N_{jt}] \end{aligned} \tag{10}$$

Then we introduce the following moral hazard/costly enforcement problem: at the beginning of the period the banker can choose to divert the fraction λ_t of available funds from the project and instead transfer them back to the household of which he or she is a member. The cost to the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction $1-\lambda_t$ of assets. However, it is too expensive for the depositors to recover the fraction λ_t the funds the banker diverted. The variable λ_t represents a financial shock that captures the degree of friction in the credit market. As I will show later, this shock directly influences leverage ratios and, consequently, the credit supply of financial intermediaries. Then we can get the following incentive constraint:

$$V_{jt} \geq \lambda_t(Q_{1t}S_{1jt} + Q_{2t}S_{2jt}) \tag{11}$$

i.e. the lost by diverting a fraction of assets should be greater than the gain from doing so.

We can express V_{jt} as follows:

$$V_{jt} = \nu_{1t}Q_{1t}S_{1jt} + \nu_{2t}Q_{2t}S_{2jt} + \eta_tN_{jt} \tag{12}$$

with

$$\nu_{1t} = E_t\{(1-\theta)\beta\Lambda_{t,t+1}(R_{1kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{1t,t+1}\nu_{1t+1}\} \tag{13}$$

$$\nu_{2t} = E_t\{(1-\theta)\beta\Lambda_{t,t+1}(R_{2kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta x_{2t,t+1}\nu_{2t+1}\} \tag{14}$$

$$\eta_t = E_t\{(1-\theta) + \beta\Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}\} \tag{15}$$

where $x_{it,t+1} \equiv Q_{it+1}S_{ijt+1}/Q_{it}S_{ijt}$ is the gross growth rate in banks' assets in sector i between t and $t+1$, and $z_{t,t+1} \equiv N_{jt+1}/N_{jt}$ is the gross growth rate of net worth. The variable ν_{it} has the interpretation of the expected discounted marginal gain to the banker of expanding assets $Q_{it}S_{ijt}$ by a unit, holding net worth N_{jt} constant, and η_t is the expected discounted value of having another unit of N_{jt}, holding S_{jt} constant.

Then the incentive constraint becomes

$$\nu_{1t}Q_{1t}S_{1jt} + \nu_{2t}Q_{2t}S_{2jt} + \eta_t N_{jt} \geq \lambda_t(Q_{1t}S_{1jt} + Q_{2t}S_{2jt}) \tag{16}$$

When this constraint binds, we have

$$\begin{aligned} N_{jt} &= \frac{\lambda_t - \nu_{1t}}{\eta_t}Q_{1t}S_{1jt} + \frac{\lambda_t - \nu_{2t}}{\eta_t}Q_{2t}S_{2jt} \\ &= [\frac{\lambda_t - \nu_{1t}}{\eta_t}\kappa_{jt} + \frac{\lambda_t - \nu_{2t}}{\eta_t}(1-\kappa_{jt})]Q_t S_{jt} \end{aligned} \tag{17}$$

Assuming that κ_{jt} is homogeneous across all banks, which is $\kappa_t = \frac{Q_{1t}S_{1t}}{Q_t S_t}$, then by aggregation, we have

$$N_t = [\frac{\lambda_t - \nu_{1t}}{\eta_t}\kappa_t + \frac{\lambda_t - \nu_{2t}}{\eta_t}(1-\kappa_t)]Q_t S_t \tag{18}$$

Define the weighted leverage ratio of the entire financial intermediaries as ϕ_t, determined by the two leverage ratios for both sectors and the portfolio weight κ_t, so that

$$Q_{1t}S_{1t} + Q_{2t}S_{2t} = Q_t S_t = \phi_t N_t \tag{19}$$

where

$$\phi_t = \frac{\eta_t}{\lambda_t - (\nu_{1t}\kappa_t + \nu_{2t}(1-\kappa_t))} \tag{20}$$

Here, $\nu_{1t}\kappa_t + \nu_{2t}(1-\kappa_t)$ represents the weighted average of the expected discounted marginal gain to the banker of expanding each kind of asset. Thus, we can capture the heterogeneous leverage ratios in two sectors, resulting from their different returns, and also feature the weighted average level of leverage ratio. Before the crisis, because of the higher return in housing market, the expected discounted marginal gain to the banker of expanding assets $Q_t S_{1jt}$ by one unit, ν_{1t}, is higher, leading to a relatively higher leverage ratio for this sector. This is like a positive feedback loop that the more returns that housing sector could bring, the higher leverage ratio for this sector would be, and the more funds the banks would provide for this sector. This positive feedback loop could also capture the procyclicality of leverage ratio documented in Adrian, Colla, and Shin (2013).

We can express the evolution of the banker's net worth as

$$N_{t+1} = \{[(R_{1kt+1} - R_{t+1})\kappa_t + (R_{2kt+1} - R_{t+1})(1-\kappa_t)]\phi_t + R_{t+1}\}N_t \tag{21}$$

And it follows that

$$z_{t,t+1} = N_{t+1}/N_t = [(R_{1kt+1} - R_{t+1})\kappa_t + (R_{2kt+1} - R_{t+1})(1-\kappa_t)]\phi_t + R_{t+1} \tag{22}$$

$$x_{1t,t+1} = Q_{1t+1}S_{1t+1}/Q_{1t}S_{1t} = \frac{\kappa_{t+1}\phi_{t+1}}{\kappa_t\phi_t}z_{t,t+1} \tag{23}$$

$$x_{2t,t+1} = Q_{2t+1}S_{2t+1}/Q_{2t}S_{2t} = \frac{(1-\kappa_{t+1})\phi_{t+1}}{(1-\kappa_t)\phi_t}z_{t,t+1} \tag{24}$$

Now, considering that N_t is the sum of the net worth of existing banker/intermediaries, N_{et}, and the net worth of entering (or "new") bankers, N_{nt}. Since the fraction θ of bankers at $t-1$ survive until t, N_{et} is given by

$$N_{et} = \theta\{[(R_{1kt} - R_t)\kappa_{t-1} + (R_{2kt} - R_t)(1-\kappa_{t-1})]\phi_{t-1} + R_t\}N_{t-1} \tag{25}$$

How much the household thinks that its new bankers need to start, depends on the scale of the assets that the exiting bankers have been intermediating. Given that the exit probability is i.i.d., the total assets of the final period of the exiting bankers at t is $(1-\theta)Q_t S_{t-1}$. Assume that each period the household transfers the fraction $\omega/(1-\theta)$ of this value to its entering bankers. Thus, in aggregate,

$$N_{nt} = \omega Q_t S_{t-1} \tag{26}$$

and

$$
\begin{aligned}
N_t &= N_{et} + N_{nt} \\
&= \theta\{[(R_{1kt} - R_t)\kappa_{t-1} + (R_{2kt} - R_t)(1 - \kappa_{t-1})]\phi_{t-1} + R_t\}N_{t-1} + \omega Q_t S_{t-1} \\
&= \theta\{[(R_{1kt} - R_t)\kappa_{t-1} + (R_{2kt} - R_t)(1 - \kappa_{t-1})]\phi_{t-1} + R_t\}N_{t-1} \\
&\quad + \omega(Q_{1t}S_{1t-1} + Q_{2t}S_{2t-1})
\end{aligned} \tag{27}
$$

The most important assumption in this two-sector model is to restrict the banks from concentrating their assets in one sector. This is achieved by controlling the banks' risks by applying an upper bound to their weighted average variance. More specifically, banks' risks are determined by the heterogeneous standard deviations σ_1 and σ_2 of the capital quality shock in each sector, and weighted by the portfolios they hold represented by κ_t. The constraint can be given by

$$
\kappa_t\sigma_1^2 + (1 - \kappa_t)\sigma_2^2 \leq \bar{\sigma}_t^2 \tag{28}
$$

where $\bar{\sigma}_t$ is the financial regulation shock which reflects the upper bound restricting the total risks and characterizing the intense of regulation. This constraint reflects the common restrictions on the weighted risks required by the Basel Accord. With this constraint, even with relatively high returns in one sector, like housing sector, banks cannot invest all their funds in this single sector, because of the higher volatility and the potential risks.

2.3 Intermediate goods firms

In the production and investment side of the economy, competitive non-financial firms produce intermediate goods that are eventually sold to retail firms. In each period, the firm in sector i finances its capital acquisition, K_{it+1}, by obtaining funds from intermediaries (without any frictions), where it issues $Q_{it}S_{it}$ amount of value of the claims against this capital, so that

$$
Q_{1t}K_{1t+1} = Q_{1t}S_{1t} \tag{29}
$$

$$
Q_{2t}K_{2t+1} = Q_{2t}S_{2t} \tag{30}
$$

Then, assuming the intermediate goods are produced using three inputs: K_{1t}, K_{2t}, and labor L_t:

$$
Y_t = w^{\alpha-1}U_t^{\alpha}[w^{\frac{1}{\sigma}}(\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]^{\frac{\alpha\sigma}{\sigma-1}}(A_tL_t)^{1-\alpha} \tag{31}
$$

where following Miao and Wang (2014), I use $\sigma > 0$ to represent the elasticity of substitution between the two types of capital, and use $w \in (0, 1)$ as a share parameter. The functional form of the production function has a constant elasticity of substitution between the two types of capital. A_t is the labor-augmenting TFP shock. ξ_{it} denotes the quality of capital in sector i so that $\xi_{it}K_{it}$ is the effective quantity of capital at time t. U_t is the utilization rate of the aggregate capital.

Let P_{mt} be the price of intermediate goods output. Assume further that the replacement price of used capital is fixed at unity. The intermediate good producers behave competitively, and each of them solves the following problem:

$$
\begin{aligned}
\max_{U_t, L_t, K_{1t}, K_{2t}} \; & P_{mt}Y_t - W_tL_t - R_{1kt}Q_{1t-1}K_{1t} - R_{2kt}Q_{2t-1}K_{2t} \\
& + (Q_{1t} - \delta(U_t))\xi_{1t}K_{1t} + (Q_{2t} - \delta(U_t))\xi_{2t}K_{2t}
\end{aligned} \tag{32}
$$

The first order conditions with respect to U_t, L_t, K_{1t} and K_{2t} give:

$$
P_{mt}\alpha\frac{Y_t}{U_t} = \delta'(U_t)(\xi_{1t}K_{1t} + \xi_{2t}K_{2t}) \tag{33}
$$

$$
P_{mt}(1-\alpha)\frac{Y_t}{L_t} = W_t \tag{34}
$$

Given that the firm earns zero profits state by state, it simply pays out the ex post return to capital to the intermediary. Accordingly, R_{ikt} is given by

$$
R_{1kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{Y_t(\xi_{1t}K_{1t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}}(\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{1t} - \delta(U_t) \right\} \frac{\xi_{1t}}{Q_{1t-1}} \tag{35}
$$

$$
R_{2kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{Y_t(\xi_{2t}K_{2t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}}(\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{2t} - \delta(U_t) \right\} \frac{\xi_{2t}}{Q_{2t-1}} \tag{36}
$$

2.4 Capital producing firms

There are two sectors producing different types of capital. At the end of period t, competitive capital producing firms buy capital from intermediate goods producing firms and then repair depreciated capital and build new capital. They then sell both the new and re-furbished capital. The cost of replacing worn out capital is unity, and the value of a unit of new capital is Q_{it}. Suppose that there are flow adjustment costs associated with producing new capital. The discounted profits for each capital producer are given by

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Lambda_{t,\tau} \left\{ (Q_{i\tau}-1)I_{in\tau} - f\left(\frac{I_{in\tau}}{I_{in\tau-1}}\right) I_{in\tau} \right\} \tag{37}$$

with

$$I_{int} \equiv I_{it} - \delta(U_t)\xi_{it}K_{it} \tag{38}$$

being the net capital created, and I_t being the gross capital created. $f(1) = f'(1) = 0$ and $f''(1) > 0$. The first order condition for investment gives the asset price:

$$Q_{it} = 1 + f\left(\frac{I_{int}}{I_{int-1}}\right) + \frac{I_{int}}{I_{int-1}} f'\left(\frac{I_{int}}{I_{int-1}}\right) - E_t\beta\Lambda_{t,t+1}\left(\frac{I_{int+1}}{I_{int}}\right)^2 f'\left(\frac{I_{int}}{I_{int-1}}\right) \tag{39}$$

2.5 Retail firms

Final output Y_t is a CES composite of a continuum of mass unity of differentiated retail firms, that use intermediate output as the sole input. The final output composite is given by

$$Y_t = \left[\int_0^1 Y_{ft}^{(\varepsilon-1)/\varepsilon} df\right]^{\varepsilon/(\varepsilon-1)} \tag{40}$$

where Y_{ft} is output by retailer f.

From cost minimization by users of final output:

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\varepsilon} Y_t \tag{41}$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\varepsilon} df\right]^{1/(1-\varepsilon)} \tag{42}$$

introduce nominal rigidities following CEE. In particular, each period a firm is able to freely adjust its price with probability $1-\gamma$, In between these periods, the firm is able to index its price to the lagged rate of inflation. Thus, the retailers' pricing problem then is to choose the optimal reset price P_t^* to solve

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^* \pi^i}{P_{t+i}} - P_{mt+i}\right] Y_{ft+i} \tag{43}$$

The first order condition gives that

$$p_t^* \equiv \frac{P_t^*}{P_t} = \mu \frac{\Gamma_t^a}{\Gamma_t^b} \tag{44}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$, and Γ_t^a and Γ_t^b are defined as

$$\Gamma_t^a = \rho_t P_{mt} Y_t + \beta\gamma E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\varepsilon} \Gamma_{t+1}^a \tag{45}$$

and

$$\Gamma_t^b = \rho_t Y_t + \beta\gamma E_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\varepsilon-1} \Gamma_{t+1}^b \tag{46}$$

It follows from (42) and calvo price setting that

$$1 = \left[\gamma\left(\frac{\pi}{\pi_t}\right)^{1-\varepsilon} + (1-\gamma) p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{47}$$

If we define the price dispersion:

$$\Delta_t = \int (P_{ft}/P_t)^{-\varepsilon} df \tag{48}$$

then it satisfies the following recursive condition:

$$\Delta_t = (1-\gamma)p_t^{*-\varepsilon} + \gamma \left(\frac{\pi}{\pi_t}\right)^{-\varepsilon} \Delta_{t-1} \tag{49}$$

Aggregating (40) yields aggregate output as

$$Y_t = \frac{1}{\Delta_t} w^{\alpha-1} U_t^{\alpha} [w^{\frac{1}{\sigma}} (\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]^{\frac{\alpha\sigma}{\sigma-1}} (A_t L_t)^{1-\alpha} \tag{50}$$

Then (33), (34), (35), and (36) become:

$$P_{mt}\alpha \frac{\Delta_t Y_t}{U_t} = \delta'(U_t)(\xi_{1t}K_{1t} + \xi_{2t}K_{2t}) \tag{51}$$

$$P_{mt}(1-\alpha)\frac{\Delta_t Y_t}{L_t} = W_t \tag{52}$$

$$R_{1kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{\Delta_t Y_t (\xi_{1t}K_{1t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}} (\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{1t} - \delta(U_t) \right\} \frac{\xi_{1t}}{Q_{1t-1}} \tag{53}$$

$$R_{2kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{\Delta_t Y_t (\xi_{2t}K_{2t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}} (\xi_{1t}K_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}K_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{2t} - \delta(U_t) \right\} \frac{\xi_{2t}}{Q_{2t-1}} \tag{54}$$

2.6 Resource constraint and government policy

Output is divided between consumption and investment. The economy-wide resource constraint is given by

$$Y_t = C_t + I_{1t} + I_{2t} + f\left(\frac{I_{1nt}}{I_{1nt-1}}\right) I_{1nt} + f\left(\frac{I_{2nt}}{I_{2nt-1}}\right) I_{2nt} \tag{55}$$

where capital evolves according to

$$K_{1t+1} = \xi_{1t}K_{1t} + I_{1nt} \tag{56}$$

$$K_{2t+1} = \xi_{2t}K_{2t} + I_{2nt} \tag{57}$$

Suppose that monetary policy is characterized by a simple Taylor rule with interest-rate smoothing::

$$i_t = (1-\rho)\left[i + \kappa_\pi \pi_t + \kappa_y (\log Y_t - \log Y_{t-1})\right] + \rho i_{t-1} \tag{58}$$

where i_t is the nominal inflation rate, i is its steady-state level, and ε_{mt} is an exogenous shock to monetary policy. The smoothing parameter ρ lies between zero and unity.

The Fisher relation gives:

$$1 + i_t = R_{t+1}E_t\pi_{t+1} \tag{59}$$

This completes the description of the model. A competitive equilibrium consists of stochastic processes of 35 aggregate endogenous variables: $\{\rho_t, W_t, L_t, C_t, R_t, N_t, \phi_t,$ $\eta_t, \nu_{1t}, \nu_{2t}, \kappa_t, z_{t-1,t}, R_{1kt}, R_{2kt}, x_{1t-1,t}, x_{2t-1,t}, Q_{1t}, Q_{2t}, S_{1t}, S_{2t}, K_{1t}, K_{2t}, Y_t, U_t, P_{mt}, I_{1nt},$ $I_{2nt}, p_t^*, \Gamma_t^a, \Gamma_T^b, \Delta_t, \pi_t, i_t, I_{1t}, I_{2t}\}$, such that 35 equations: (3), (4), (5), (19), (20), (22), (23), (24), (13), (14), (15), (27), (28), (29), (30), (50), (51), (52), (53), (54), (38) for $i = 1,\ 2$, (39) for $i = 1,\ 2$, (44), (45), (46), (47), (49), (55), (56), (57), (58), (59) and

$$\frac{Q_{1t}S_{1t}}{Q_{1t}S_{1t}} = \frac{\kappa_t}{1-\kappa_t} \tag{60}$$

hold, with 6 exogenous variables $\{A_t, \chi_t, \lambda_t, \bar{\sigma}_t^2, \xi_{1t}, \xi_{2t}\}$.

3 Detrend Transformation

Since the model has a unit root, the TFP shock A_t, we have to appropriately transform the equilibrium system into a stationary one. First, I define the following transformed variables:

$$\tilde{\rho}_t = \rho_t \cdot A_t,\ \tilde{W}_t = W_t/A_t,\ \tilde{C}_t = C_t/A_t,\ \tilde{S}_{it} = S_{it}/A_t,\ \tilde{N}_t = N_t/A_t,$$
$$\tilde{K}_{it} = K_{it}/A_t,\ \tilde{Y}_t = Y_t/A_t,\ \tilde{I}_{int} = I_{int}/A_t,\ \tilde{I}_{it} = I_{it}/A_t.$$

The other variables are stationary and there is no need to scale them. The transformed equilibrium is a system of 35 equations for 35 transformed variables: $\{\tilde{\rho}_t, \tilde{W}_t, L_t, \tilde{C}_t, R_t, \tilde{N}_t, \phi_t, \eta_t, \nu_{1t}, \nu_{2t}, \kappa_t, z_{t-1,t}, R_{1kt}, R_{2kt}, x_{1t-1,t}, x_{2t-1,t}, Q_{1t}, Q_{2t}, \tilde{S}_{1t}, \tilde{S}_{2t}, \tilde{K}_{1t}, \tilde{K}_{2t}, \tilde{Y}_t, U_t, P_{mt}, \tilde{I}_{1nt}, \tilde{I}_{2nt}, p^*_t, \Gamma^a_t, \Gamma^b_T, \Delta_t, \pi_t, i_t, \tilde{I}_{1t}, \tilde{I}_{2t}\}$. The detrended system is listed in the Appendix B.

4 Calibration

The calibrated parameters are listed in Table 1, Table 2 and Table 3.

Table 1: Calibrated Parameters for Two-sector Model

Parameter	Values	Description
α	0.33	Capital share in production
β	0.99	Subjective discounting factor
$\delta(1)$	0.025	Steady-state depreciation rate
$\bar{g}_A$	1.0079	TFP growth rate
π	1.007725	Inflation rate
γ	0.9608	Price adjustment probability
h	0.5153	Habit formation parameter
φ	0.263	Inverse Frisch elasticity of labor supply
λ	0.2807	Fraction of capital that can be diverted
η	1.3549	Value of expanding net worth
$\frac{\delta''(1)}{\delta'(1)}$	9.1131	Capacity utilization parameter
ξ_1	1	Mean value of capital quality shock in sector 1
ξ_2	1	Mean value of capital quality shock in sector 2
$f(\bar{g}_A)$	0.7420	Flow adjustment costs coefficient
$f'(\bar{g}_A)$	1.4972	Flow adjustment costs coefficient
$f''(\bar{g}_A)$	1.2389	Flow adjustment costs coefficient
$\frac{\tilde{C}}{\tilde{Y}}$	0.7	Consumption GDP ratio
ρ	0.3214	Smoothing parameter of the Taylor rule
κ_π	1.5807	Inflation coefficient of the Taylor rule
κ_y	0.0820	Output gap coefficient of the Taylor rule
θ	0.9855	survival rate of the bankers
$\nu 1$	0.1	Value of expanding assets in capital 1
$\nu 2$	0.1	Value of expanding assets in capital 2
R_{1k}	1.05	Return to capital 1
R_{2k}	1.02	Return to capital 2

Due to the log-linearization solution method, I do not need to parameterize the depreciation function $\delta(\cdot)$. Only need to know the steady-state values of $\delta(1)$, $\delta'(1)$ and $\delta''(1)$. The values for parameters $\{\alpha, \beta, \delta(1), \bar{g}_A, \pi\}$ are calibrated from literature or by matching the US data. Then I estimate the values for parameters $\{\varphi, \gamma, h, \eta, \frac{\delta''(1)}{\delta'(1)}, \xi_1, \xi_2, f(\bar{g}_A), f'(\bar{g}_A), f''(\bar{g}_A), \rho, \kappa_\pi, \kappa_y, \theta, \nu_1, \nu_2, \rho_A, \rho_\chi, \rho_{\xi_1}, \rho_{\xi_2}, \rho_\lambda, \sigma_A, \sigma_\chi, \sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_\lambda, \sigma_1^2, \sigma_2^2\}$ using the Bayesian estimation. I assume the standard deviation of capital quality shock in sector 1, σ_{ξ_1} is ten times as large as that in sector 2, σ_{ξ_2} to capture the high risk in sector 1. Higher volatility of the shocks results in higher risks in sector 1. Thus, I assume $\sigma_1^2 = \sigma_{\xi_1}^2$, and

Table 2: Calibrated Parameters for Two-sector Model (continued)

Parameter	Values	Description
$\frac{\tilde{I}_1}{\tilde{Y}}$	0.1	Investment to GDP ratio in sector 1
$\frac{\tilde{I}_2}{\tilde{Y}}$	0.1	Investment to GDP ratio in sector 2
$\frac{\tilde{K}_1}{\tilde{Y}}$	0.45	Capital to GDP ratio in sector 1
$\frac{\tilde{K}_2}{\tilde{Y}}$	0.45	Capital to GDP ratio in sector 2
σ_1^2	0.0303	Volatility/risk in sector 1
σ_2^2	0.000303	Volatility/risk in sector 2

Table 3: Calibrated Parameters for shocks in Two-sector Model

Parameter	Values	Description
ρ_A	0.9743	AR(1) coefficient for TFP shock
ρ_χ	0.9974	AR(1) coefficient for labor supply shock
ρ_{ξ_1}	0.7316	AR(1) coefficient for capital quality shock in sector 1
ρ_{ξ_2}	0.7316	AR(1) coefficient for capital quality shock in sector 2
$\rho_{\bar{\sigma}}$	0.95	AR(1) coefficient for financial regulation shock
ρ_λ	0.9812	AR(1) coefficient for financial shock
σ_A	0.0027	Std. for TFP shock
σ_χ	0.0577	Std. for labor supply shock
σ_{ξ_1}	0.174	Std. for capital quality shock in sector 1
σ_{ξ_2}	0.0174	Std. for capital quality shock in sector 2
$\sigma_{\bar{\sigma}}$	0.01	Std. for financial regulation shock
σ_λ	2.0432	Std. for financial shock

$\sigma_2^2 = \sigma_{\xi_2}^2$. These are the default values I set for baseline case. Later, I will carry out experiments with different value combinations of $\{\sigma_{\xi_1}, \sigma_{\xi_2}, \sigma_1^2, \sigma_2^2\}$. In each case, I maintain the assumption that $\sigma_1^2 = \sigma_{\xi_1}^2$ and $\sigma_2^2 = \sigma_{\xi_2}^2$. Finally, I calibrate the parameters $\{\frac{\tilde{I}_1}{\tilde{Y}}, \frac{\tilde{I}_2}{\tilde{Y}}, \frac{\tilde{K}_1}{\tilde{Y}}, \frac{\tilde{K}_2}{\tilde{Y}}, R_{1k}, R_{2k}\}$ such that they roughly match the magnitude in U.S. data. Since in U.S. data, the investment share is around 20%, and the consumption share is around 70%, assuming there are only these two sectors in the economy, I set $\frac{\tilde{I}_1}{\tilde{Y}} = \frac{\tilde{I}_2}{\tilde{Y}} = 0.1$, and $\frac{\tilde{C}}{\tilde{Y}} = 0.7$. Similarly, I set $\frac{\tilde{K}_1}{\tilde{Y}} = \frac{\tilde{K}_2}{\tilde{Y}} = 0.45$, since the capital/GDP ratio ranges from 0.5 to 1.5. Higher risk means higher returns. Thus, with R_k estimated to be around 1.0186 in last section, I set $R_{1k} = 1.05$ and $R_{2k} = 1.02$.

Other than these parameters, I manually pick different values for σ to control the elasticity of substitution between the two types of capital, which represents the connection between these two sectors, and choose different values for w to control the relative capital share of the two sectors utilized in production. This can help me capture how different levels of connection between two sectors, and how different relative capital shares in the production function would affect the shock transmission mechanisms that I am particularly interested in. I also pick different values for $\sigma_1^2 = \sigma_{\xi_1}^2$ and $\sigma_2^2 = \sigma_{\xi_2}^2$ to capture different combinations of the volatility of the two sectors. With experiments conducted with different variance combinations, I would be able to study how they affect the shock propagation from one sector to the other. In addition, the mean value of financial regulation parameter $\bar{\sigma}^2$ should be adjusted to accommodate different variance combinations. Another question I am interested in is how the financial regulation shock characterized by the AR(1) process, with AR(1) coefficient $\rho_{\bar{\sigma}}$ and standard error $\bar{\sigma}_t$, would affect the economy.

These experiments are carried out in the following sections. In the baseline case, I assume these parameters take the values as listed in the first column in table 6. I use the baseline parameter values to analyze the effects of financial shock λ_t and financial regulation shock $\bar{\sigma}_t$. Then I adjust the values for other parameters to analyze the capital quality shock propagation. Parameter values for all the other cases are also listed.

5 Quantitative Experiments

In this section, I first conduct experiments to examine the effects of a financial shock λ_t and a financial regulation shock $\bar{\sigma}_t$ on the economy. These experiments capture how disruptions within the financial system transmit to the broader economy. I then analyze how a capital quality shock originating in one sector propagates through the financial system and spills over into the other sector.

Table 4: Parameter Values in Experiments

Parameter	Baseline	1	2	3
σ	0.1	0.1	3	0.1
w	0.5	0.5	0.5	0.5
σ_1^2	0.0303	1	0.0303	0.0303
σ_2^2	0.000303	0.0001	0.000303	0.000303
$\bar{\sigma}^2$	0.02	0.5	0.02	0.02
σ_{ξ_1}	0.174	1	0.174	0.174
σ_{ξ_2}	0.0174	0.01	0.0174	0.0174
$\rho_{\bar{\sigma}}$	0.95	0.95	0.95	0.5
$\sigma_{\bar{\sigma}}$	0.01	0.01	0.01	0.1

5.1 Financial shock λ_t and financial regulation shock $\bar{\sigma}_t$

Intuitively, a positive financial shock, representing a greater ability for banks to divert assets to their owners, tightens the lending constraint and reduces the leverage ratio. As a result, credit supply contracts, potentially triggering an economic downturn.

When a negative financial regulation shock occurs, it affects the economy through two distinct channels. First, tighter risk constraints force the bank to reduce its holdings of riskier assets, namely, those in sector 1, and reallocate credit toward safer assets in sector 2. This reallocation channel results in an increase in capital stock in sector 2. However, by holding fewer high-return assets, the bank's overall profitability declines, leading to a reduction in its net worth. This, in turn, tightens the bank's borrowing constraint and reduces total credit supply to the economy, including to sector 2. This constitutes the net worth channel, which has a contractionary effect on capital accumulation. The overall impact on sector 2 depends on which channel dominates, and this is determined by the underlying parameter values in the model.

The baseline case with $\sigma = 0.1$ and $w = 0.5$ implies a relatively low elasticity of substitution between the two types of capital, while capital from both sectors contributes equally to production due to the equal capital share.

Figure 3 displays the impulse responses of key macroeconomic and financial variables to a one-standard-deviation financial shock λ_t. The shock leads to a pronounced downturn in output ($\hat{Y}_t$), hours worked ($\hat{L}_t$), and capital stocks in both sectors ($\hat{K}_{1t}$ and $\hat{K}_{2t}$), indicating a broad-based contraction. The decline in the leverage ratio and rise in the credit spread reflect a tightening in overall credit conditions, as banks reduce lending in response to the shock.

Interestingly, despite the contraction in credit supply, bank net worth ($\hat{N}_t$) increases in the short run due to the widening spread, which improves the return on remaining assets. This highlights two opposing forces influencing banks' ability to supply credit: on one hand, the rising spread boosts profitability and raises net worth; on the other hand, the tighter leverage constraint reduces banks' capacity to lend. In this case, the decline in leverage dominates the rise in net worth, resulting in an overall reduction in credit supply and a contraction in economic activity.

Figure 4 presents the impulse responses of the same key macro-financial variables to a one-standard-deviation financial regulation shock ($\bar{\sigma}_t$). A negative shock to financial regulation tightens the bank's

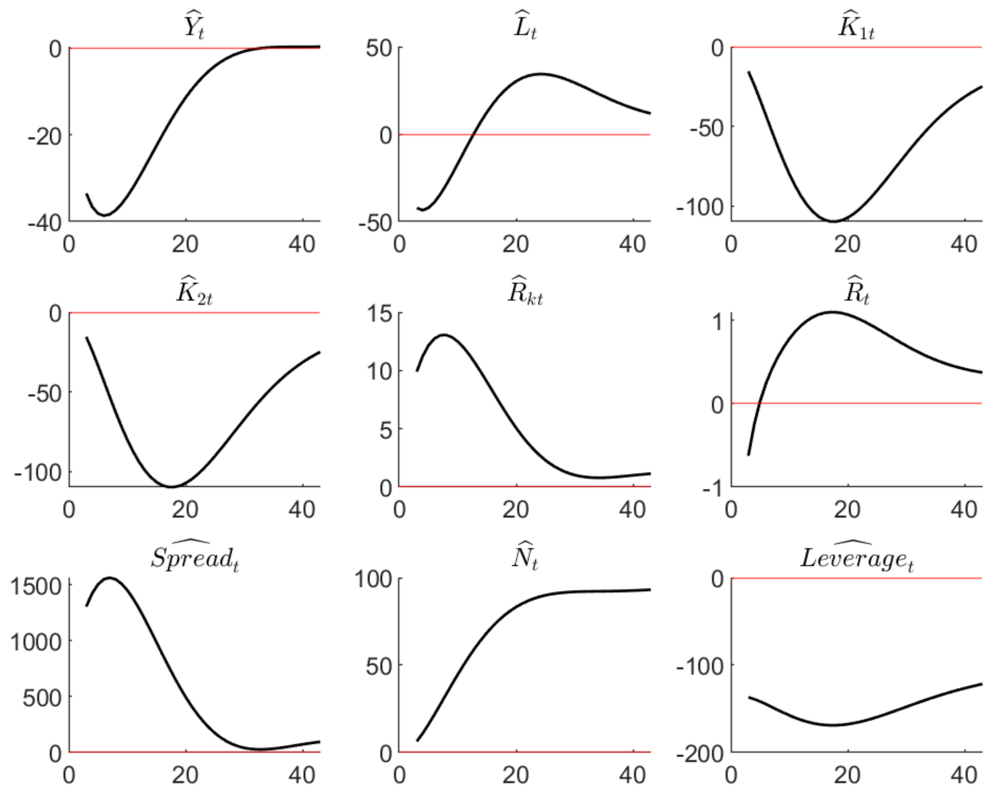


Figure 3: Impulse responses to a one-standard-deviation financial shock (λ_t) with $\sigma = 0.1$ and $w = 0.5$

risk-taking constraint, forcing the bank to reduce its exposure to riskier assets, specifically, those in sector 1. Consequently, the bank cuts its holdings of sector 1 capital and reduces its overall leverage.

Although this shock initially encourages a reallocation of credit toward safer assets in sector 2, the total credit supply contracts due to a decline in bank net worth. As a result, capital stock in sector 2 also falls. This outcome indicates that the second channel, a contraction in total credit due to declining net worth, dominates the first channel of credit reallocation. The result is a broad-based decline in output, employment, and capital accumulation across both sectors.

Figure 5 illustrates the impulse responses to a one-standard-deviation financial regulation shock ($\bar{\sigma}_t$) under an alternative calibration: the elasticity of substitution between capital types is increased to $\sigma = 3$, and the capital share of sector 1 is reduced to $w = 0.1$. Under these conditions, the dynamics of the economy respond quite differently compared to the baseline case.

A negative financial regulation shock, which tightens the bank's risk constraint, still leads to a reduction in credit to the riskier sector 1. However, because of the higher elasticity of substitution, capital from sector 2 becomes a more effective substitute. As a result, the bank reallocates credit toward sector 2, which experiences a noticeable increase in capital accumulation. This substitution leads to an expansion in sector 2 that more than offsets the contraction in sector 1, allowing the economy to avoid an overall downturn.

Consequently, output, labor, and total capital exhibit much milder declines, or even slight expansions, compared to the baseline. The results indicate that in this setting, the credit reallocation channel dominates the net worth channel, with sector 2 absorbing the redirected credit and driving a modest recovery. This underscores the importance of production structure and substitution elasticity in shaping the transmission of financial regulation shocks.

Figure 6 shows the impulse responses to a one-standard-deviation financial regulation shock when the shock is less persistent ($\rho_{\bar{\sigma}} = 0.5$). In this case, the tightening of the bank's risk constraint is more short-lived. As a result, banks temporarily reduce their exposure to sector 1 and reallocate funds

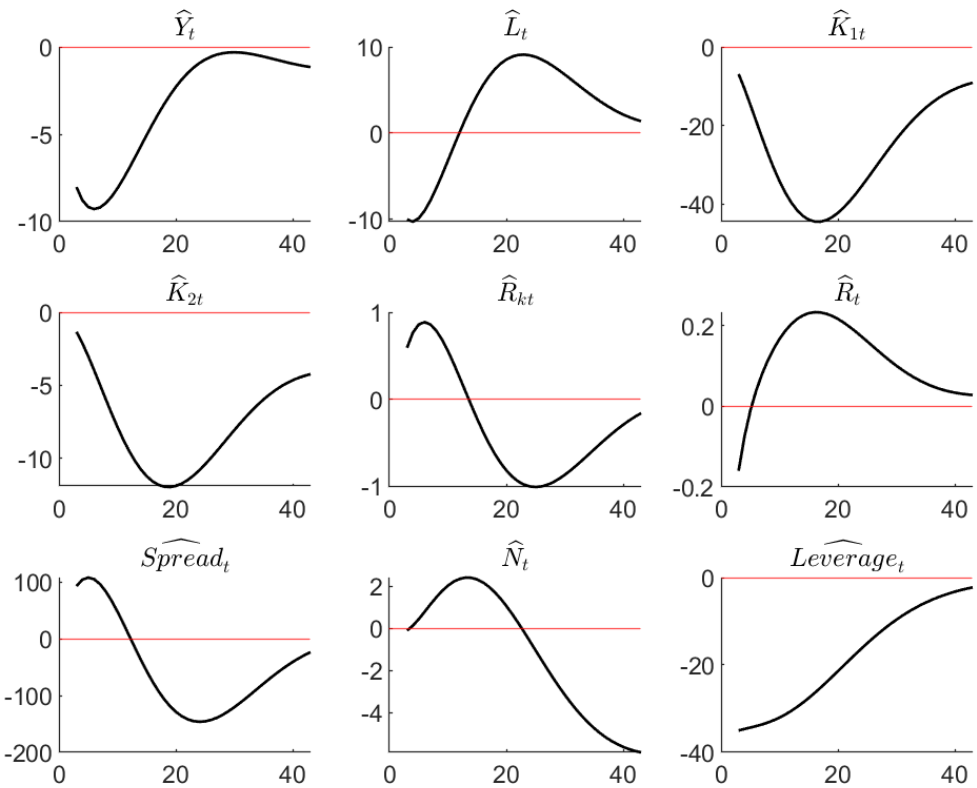


Figure 4: Impulse responses to a one-standard-deviation financial regulation shock ($\bar{\sigma}_t$) with $\sigma = 0.1$ and $w = 0.5$

toward sector 2, leading to a modest increase in sector 2's capital stock.

However, because the shock is less persistent, the overall adjustment in leverage and net worth is limited, and the reallocation effect is relatively short-lived. The improvement in sector 2 is not sustained, and the total credit supply contracts only briefly. This indicates that when financial regulation shocks are transitory, the credit reallocation channel (from sector 1 to sector 2) is still active but less pronounced, and the reduction in total credit supply becomes the more dominant force over time. The modest and temporary nature of the changes in macro variables such as output, labor, and capital further reflects the muted transmission of short-lived regulatory shocks.

Compared to the more persistent case in Figure 4, where the regulatory constraint persists and imposes a sustained drag on both sectors, the short-lived nature of the shock in Figure 6 limits its long-run impact. In Figure 4, the reduction in total credit supply dominates, leading to a broader downturn across sectors. In contrast, Figure 6 reflects a muted transmission: the credit reallocation channel initially provides support to sector 2, and the total credit contraction is less severe and quickly reversed.

This comparison underscores the importance of shock persistence in determining the dominant transmission channel. When regulation tightens temporarily, its effects on leverage, spreads, and credit allocation are more contained, and the macroeconomic response is correspondingly modest.

5.2 Capital quality shocks ξ_{1t}

This section investigates how adverse developments in sector 1, such as a capital quality shock, transmit to the rest of the economy. Specifically, it explores the mechanisms through which sector-specific disruptions affect sector 2 and aggregate outcomes, including output, capital allocation, and credit conditions.

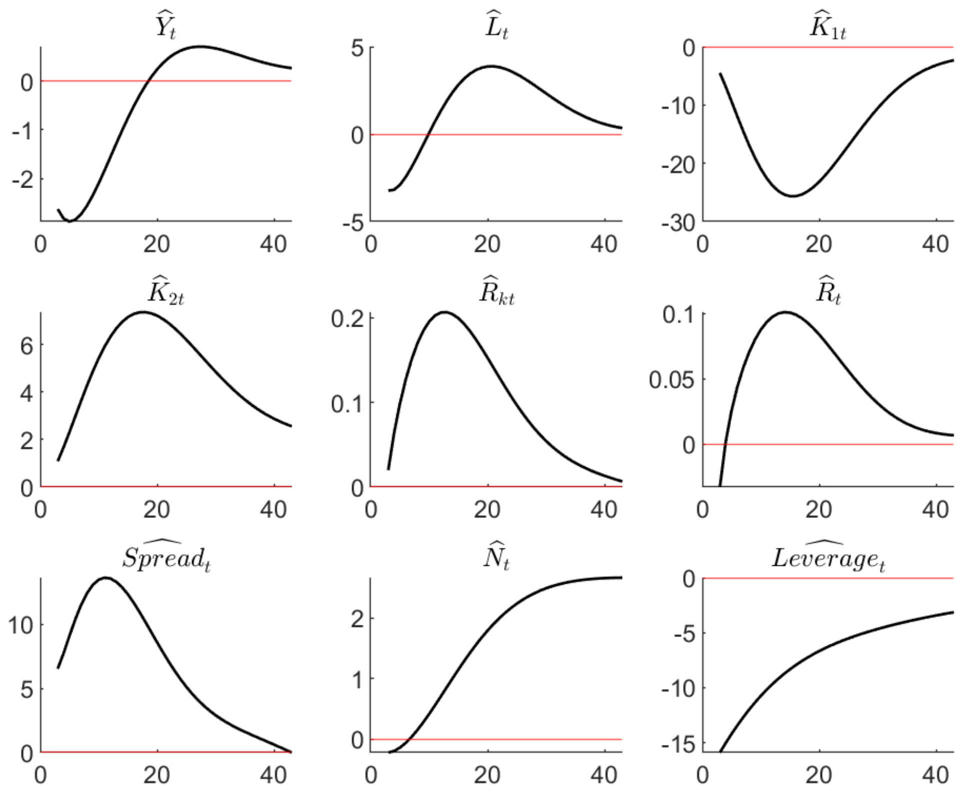


Figure 5: Impulse responses to a one-standard-deviation financial regulation shock ($\bar{\sigma}_t$) with $\sigma = 3$ and $w = 0.5$

Figure 7 illustrates the impulse responses to a one-standard-deviation negative capital quality shock originating in sector 1 under the baseline calibration ($\sigma = 0.1$, $w = 0.5$). As expected, the deterioration in sector 1 significantly weakens the bank's balance sheet by reducing the value of its asset holdings. The decline in bank net worth leads to a contraction in credit supply, which in turn affects sector 2 by limiting its access to funding. Consequently, capital accumulation in sector 2 falls, even though it is not directly hit by the shock.

Interestingly, the risk-free interest rate rises following the shock, which raises the cost of borrowing. This induces an increase in labor supply, reflected in the rise in hours worked, as households substitute away from capital. The temporary boost in labor input contributes to an initial increase in output before the broader credit contraction drags the economy downward. Overall, the figure highlights how a sector-specific shock can spill over to the rest of the economy through financial intermediation and general equilibrium effects.

Figure 8 presents the impulse responses to a one-standard-deviation capital quality shock in sector 1 under an alternative calibration with asymmetric volatility: $\sigma_1^2 = 1$ and $\sigma_2^2 = 0.0001$. The results differ markedly from the baseline case. In this scenario, the deterioration in sector 1 leads not to a contraction, but to an expansion in sector 2's capital stock and a substantial boom in aggregate output.

This counterintuitive result arises because the bank shifts its portfolio toward the more stable sector 2. The extremely low volatility in sector 2 makes it an attractive destination for capital reallocation, especially when the risk associated with sector 1 spikes. Moreover, the bank's net worth is not significantly impaired by the shock in sector 1, due to a sharp increase in the spread, which boosts returns on existing holdings in that sector.

As a result, the credit reallocation channel dominates: the banking system effectively insulates the stable sector from the turmoil in the volatile one. This experiment highlights an important in-

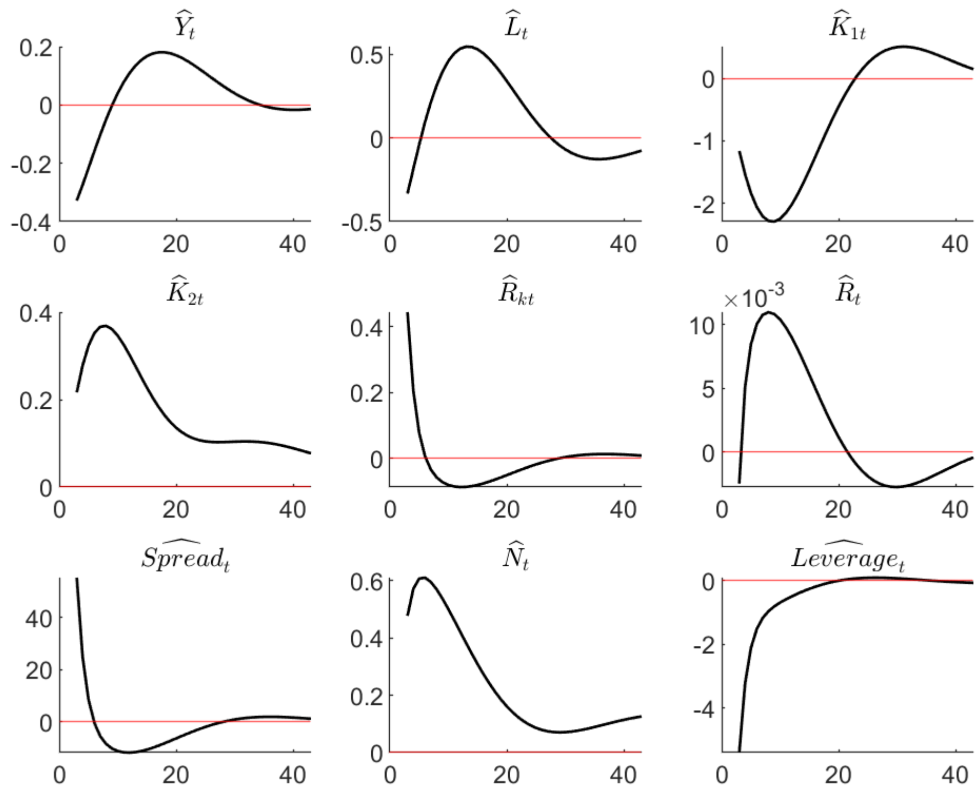


Figure 6: Impulse responses to a one-standard-deviation financial regulation shock with $\rho_{\bar{\sigma}} = 0.5$

sight—when sector 2 is highly stable, the financial system may redirect funds toward it in times of stress, enabling it to expand even amid sector-specific deterioration elsewhere.

6 Conclusion

This paper extends the Gertler-Karadi DSGE framework to a two-sector model in order to analyze the transmission of financial shocks through the credit supply channel. Motivated by the observed vulnerabilities in the housing sector during the Great Recession, the model incorporates heterogeneity in sectoral leverage and allows for capital quality shocks, financial frictions in banking, and regulatory constraints on risk-taking. This richer structure captures key features of how disruptions in one sector can spill over into others through the financial system.

Through a series of calibrated quantitative experiments, the analysis highlights the critical role of banks' balance sheet health and portfolio composition in amplifying or dampening the effects of financial shocks. In particular, the results show that financial shocks—modeled as increases in the incentive to divert funds—tighten borrowing constraints, reduce leverage, and contract credit supply, leading to downturns across both sectors. Similarly, negative financial regulation shocks restrict banks' risk exposure and shift credit away from riskier sectors, with differing consequences depending on sectoral risk-return profiles and the elasticity of substitution in production.

Importantly, the model identifies two distinct propagation channels: a credit reallocation channel, in which banks shift funds between sectors based on risk, and a net worth channel, where shocks to intermediary balance sheets contract aggregate lending. The dominance of one channel over the other depends on model parameters such as sectoral volatility, capital share, and regulation intensity. This explains why similar shocks can yield vastly different macroeconomic outcomes depending on the underlying sectoral structure and financial environment.

From a policy perspective, these findings underscore the importance of explicitly modeling the

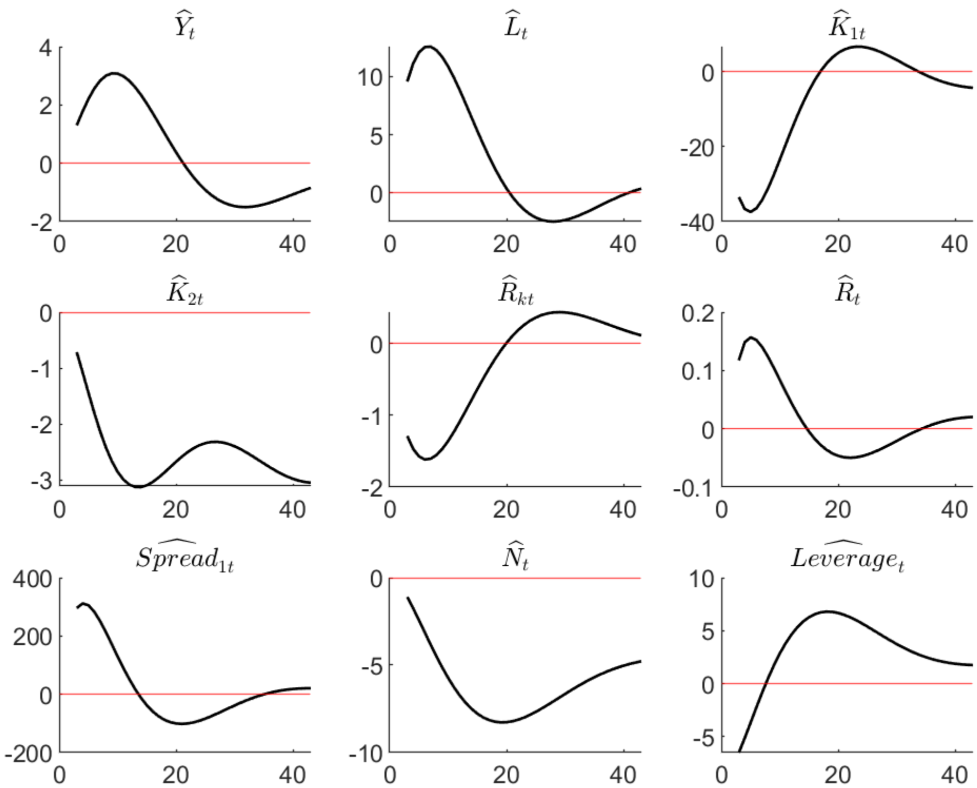


Figure 7: Impulse responses to a one-standard-deviation capital quality shock in sector 1 with $\sigma = 0.1$ and $w = 0.5$

financial intermediary sector when analyzing macro-financial linkages. They also highlight the potential for financial regulation to have unintended sectoral consequences, depending on how tightly risk is constrained. Future research could extend this framework to include richer household heterogeneity, endogenous default, or interactions with unconventional monetary policies to better understand optimal financial regulation in complex economies.

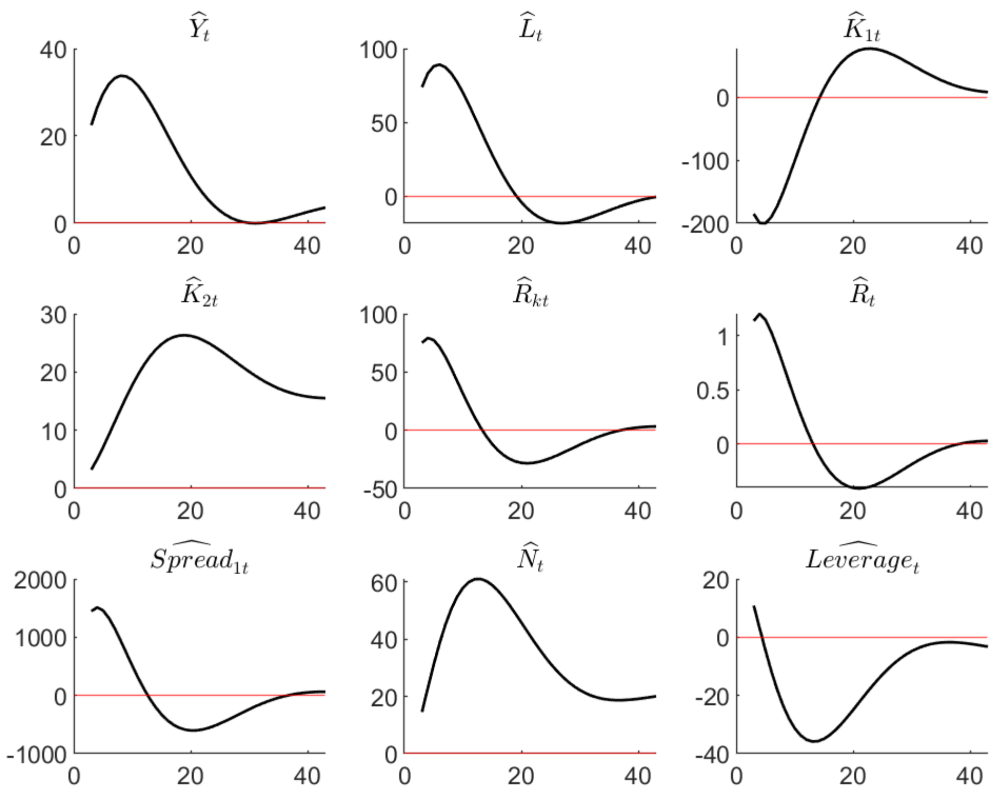


Figure 8: Impulse responses to a one-standard-deviation capital quality shock in sector 1 with $\sigma_1^2 = 1$ and $\sigma_2^2 = 0.0001$

References

Adrian, Tobias, Paolo Colla and Hyun Song Shin, 2013. "Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007 to 2009," *NBER Macroeconomics Annual*, University of Chicago Press, vol. 27(1), pages 159-214.

An, Sungbae and Frank Schorfheide, 2007. "Bayesian Analysis of DSGE Models," *Econometric Reviews*, Taylor and Francis Journals, vol. 26(2-4), pages 113-172.

Bernanke, Ben S., Mark Gertler and Simon Gilchrist, 1999. "The financial accelerator in a quantitative business cycle framework," *Handbook of Macroeconomics*, in: J. B. Taylor and M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393, Elsevier.

Brooks, Stephen and Andrew Gelman, 1998. "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, vol. 7(4), pages 434-455.

Brunnermeier, Markus K., Thomas M. Eisenbach and Yuliy Sannikov, 2012. "Macroeconomics with Financial Frictions: A Survey," NBER Working Papers 18102, National Bureau of Economic Research, Inc.

Brunnermeier, Markus K. and Yuliy Sannikov, 2014. "A Macroeconomic Model with a Financial Sector," *American Economic Review*, American Economic Association, vol. 104(2), pages 379-421, February.

Carlstrom, Charles T., Timothy S. Fuerst and Matthias Paustian, 2016. "Optimal Contracts, Aggregate Risk, and the Financial Accelerator," *American Economic Journal: Macroeconomics*, American Economic Association, vol. 8(1), pages 119-147, January.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans, 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, University of Chicago Press, vol. 113(1), pages 1-45, February.

Claessens, Stijn and Ayhan Kose, 2017. "Macroeconomic implications of financial imperfections : a survey," Policy Research Working Paper Series 8260, The World Bank.

Del Negro, Marco and Frank Schorfheide, 2008. "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics*, Elsevier, vol. 55(7), pages 1191-1208, October.

Dong, Feng, Jianjun Miao and Pengfei Wang, 2020. "Asset Bubbles and Monetary Policy," *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, vol. 37, pages 68-98, August.

Fernandez-Villaverde, Jesus, Juan F. Rubio Ramírez and Frank Schorfheide, 2016. "Solution and Estimation Methods for DSGE Models," *Handbook of Macroeconomics*, in: J. B. Taylor and Harald Uhlig (ed.), Handbook of Macroeconomics, edition 1, volume 2, chapter 0, pages 527-724, Elsevier.

Gertler, Mark and Simon Gilchrist, 2018. "What Happened: Financial Factors in the Great Recession,"
emphJournal of Economic Perspectives, American Economic Association, vol. 32(3), pages 3-30, Summer.

Gertler, Mark and Peter Karadi, 2011. "A model of unconventional monetary policy," *Journal of Monetary Economics*, Elsevier, vol. 58(1), pages 17-34, January.

Gertler, Mark and Nobuhiro Kiyotaki, 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Handbook of Monetary Economics*, in: Benjamin M. Friedman and Michael Woodford (ed.), Handbook of Monetary Economics, edition 1, volume 3, chapter 11, pages 547-599, Elsevier.

Gertler, Mark and Nobuhiro Kiyotaki, 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, American Economic Association, vol. 105(7), pages 2011-2043, July.

Gertler, Mark, Nobuhiro Kiyotaki and Andrea Prestipino, 2016. "Wholesale Banking and Bank Runs in Macroeconomic Modeling of Financial Crises," *Handbook of Macroeconomics*, in: J. B. Taylor and Harald Uhlig (ed.), Handbook of Macroeconomics, edition 1, volume 2, chapter 0, pages 1345-1425, Elsevier.

He, Zhiguo and Arvind Krishnamurthy, 2019. "A Macroeconomic Framework for Quantifying Systemic Risk," *American Economic Journal: Macroeconomics*, American Economic Association, vol. 11(4), pages 1-37, October.

Ikeda, Daisuke, 2013. "Monetary Policy and Inflation Dynamics in Asset Price Bubbles," Bank of Japan Working Paper Series 13-E-4, Bank of Japan.

Jermann, Urban and Vincenzo Quadrini, 2012. "Macroeconomic Effects of Financial Shocks," *American Economic Review*, American Economic Association, vol. 102(1), pages 238-271, February.

Justiniano, Alejandro, Giorgio E. Primiceri and Andrea Tambalotti, 2010. "Investment shocks and business cycles," *Journal of Monetary Economics*, Elsevier, vol. 57(2), pages 132-145, March.

Kiyotaki, Nobuhiro and John Moore, 1997. "Credit Cycles," *Journal of Political Economy*, University of Chicago Press, vol. 105(2), pages 211-248, April.

Liu, Zheng, Pengfei Wang and Tao Zha, 2013. "Land-Price Dynamics and Macroeconomic Fluctuations," *Econometrica*, Econometric Society, vol. 81(3), pages 1147-1184, May.

Miao, Jianjun, 2014. "Introduction to economic theory of bubbles," *Journal of Mathematical Economics*, Elsevier, vol. 53(C), pages 130-136.

Miao, Jianjun and Pengfei Wang, 2014. "Sectoral bubbles, misallocation, and endogenous growth," *Journal of Mathematical Economics*, Elsevier, vol. 53(C), pages 153-163.

Miao, Jianjun and Pengfei Wang, 2015. "Banking bubbles and financial crises," *Journal of Economic Theory*, Elsevier, vol. 157(C), pages 763-792.

Miao, Jianjun, Pengfei Wang and Zhiwei Xu, 2015. "A Bayesian DSGE Model of Stock Market Bubbles and Business Cycles," *Quantitative Economics*, Elsevier, vol. 6(3), pages 599-635.

Miao, Jianjun and Dongling Su, 2021. "Fiscal and Monetary Policy Interactions in a Model with Low Interest Rates," Working paper.

Michael Woodford, 2010. "Financial Intermediation and Macroeconomic Analysis," *Journal of Economic Perspectives*, American Economic Association, vol. 24(4), pages 21-44, Fall.

Ramey, V.A., 2016. "Macroeconomic Shocks and Their Propagation," *Handbook of Macroeconomics*, in: J. B. Taylor and Harald Uhlig (ed.), Handbook of Macroeconomics, edition 1, volume 2, chapter 0, pages 71-162, Elsevier.

Smets, Frank and Raf Wouters, 2003. "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, MIT Press, vol. 1(5), pages 1123-1175, September.

Smets, Frank and Raf Wouters, 2005. "Comparing shocks and frictions in US and euro area business cycles: a Bayesian DSGE Approach," *Journal of Applied Econometrics*, John Wiley and Sons, Ltd., vol. 20(2), pages 161-183.

Stracca, Livio and Fabio Fornari, 2013. "What does a financial shock do? First international evidence," Working Paper Series 1522, European Central Bank.

A Detrended System

1. Labor supply,

$$\tilde{\rho}_t \tilde{W}_t = \chi_t L_t^{\varphi} \tag{61}$$

2. Marginal utility for consumption,

$$\tilde{\rho}_t = (\tilde{C}_t - h\tilde{C}_{t-1}/g_{At})^{-1} - \beta h \mathbf{E}_t(\tilde{C}_{t+1} \cdot g_{At+1} - h\tilde{C}_t)^{-1} \tag{62}$$

3. Risk-free rate,

$$\mathbf{E}_t \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \frac{R_{t+1}}{g_{At+1}} = 1 \tag{63}$$

4. Total intermediary demand for assets,

$$Q_{1t}\tilde{S}_{1t} + Q_{2t}\tilde{S}_{2t} = \phi_t \tilde{N}_t \tag{64}$$

5. Leverage ratio,

$$\phi_t = \frac{\eta_t}{\lambda_t - (\nu_{1t}\kappa_t + \nu_{2t}(1-\kappa_t))} \tag{65}$$

6. Gross growth rate of net worth,

$$z_{t,t+1} = [(R_{1kt+1} - R_{t+1})\kappa_t + (R_{2kt+1} - R_{t+1})(1-\kappa_t)]\phi_t + R_{t+1} \tag{66}$$

7. Gross growth rate in assets in sector 1,

$$x_{1t,t+1} = \frac{\kappa_{t+1}\phi_{t+1}}{\kappa_t \phi_t} z_{t,t+1} \tag{67}$$

8. Gross growth rate in assets in sector 2,

$$x_{2t,t+1} = \frac{(1-\kappa_{t+1})\phi_{t+1}}{(1-\kappa_t)\phi_t} z_{t,t+1} \tag{68}$$

9. Expected discounted marginal gain to the banker of expanding assets in sector 1 by a unit, holding net worth constant,

$$\nu_{1t} = \mathbf{E}_t\{(1-\theta)\beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \frac{R_{1kt+1} - R_{t+1}}{g_{At+1}} + \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \theta \frac{x_{1t,t+1}\nu_{1t+1}}{g_{At+1}}\} \tag{69}$$

10. Expected discounted marginal gain to the banker of expanding assets in sector 1 by a unit, holding net worth constant,

$$\nu_{2t} = \mathbf{E}_t\{(1-\theta)\beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \frac{R_{2kt+1} - R_{t+1}}{g_{At+1}} + \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \theta \frac{x_{2t,t+1}\nu_{2t+1}}{g_{At+1}}\} \tag{70}$$

11. Expected discounted value of having another unit of net worth, holding assets constant,

$$\eta_t = \mathbf{E}_t\{(1-\theta) + \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} \theta \frac{z_{t,t+1}\eta_{t+1}}{g_{At+1}}\} \tag{71}$$

12. Evolution of the banker's net worth,

$$\begin{aligned} \tilde{N}_t =& \theta \left\{ [(R_{1kt} - R_t)\kappa_{t-1} + (R_{2kt} - R_t)(1-\kappa_{t-1})]\phi_{t-1} + R_t \right\} \frac{\tilde{N}_{t-1}}{g_{At}} \\ &+ \omega \left(Q_{1t} \frac{\tilde{S}_{1t-1}}{g_{At}} + Q_{2t} \frac{\tilde{S}_{2t-1}}{g_{At}} \right) \end{aligned} \tag{72}$$

13. Risk control,

$$\kappa_t \sigma_1^2 + (1-\kappa_t)\sigma_2^2 \leq \bar{\sigma}^2 \tag{73}$$

14. Firm in sector 1 financing its capital acquisition,

$$\tilde{K}_{1t+1} = \frac{\tilde{S}_{1t}}{g_{At+1}} \tag{74}$$

15. Firm in sector 2 financing its capital acquisition,

$$\tilde{K}_{2t+1} = \frac{\tilde{S}_{2t}}{g_{At+1}} \tag{75}$$

16. Aggregate output,

$$\tilde{Y}_t = \frac{1}{\Delta_t} w^{\alpha-1} U_t^{\alpha} [w^{\frac{1}{\sigma}} (\xi_{1t}\tilde{K}_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}\tilde{K}_{2t})^{\frac{\sigma-1}{\sigma}}]^{\frac{\alpha\sigma}{\sigma-1}} (A_t L_t)^{1-\alpha} \tag{76}$$

17. Utilization rate,

$$P_{mt}\alpha \frac{\Delta_t \tilde{Y}_t}{U_t} = \delta'(U_t)(\xi_{1t}\tilde{K}_{1t} + \xi_{2t}\tilde{K}_{2t}) \tag{77}$$

18. Labor demand,

$$P_{mt}(1-\alpha)\frac{\Delta_t \tilde{Y}_t}{L_t} = \tilde{W}_t \tag{78}$$

19. The capital return in sector 1,

$$R_{1kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{\Delta_t \tilde{Y}_t (\xi_{1t}\tilde{K}_{1t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}} (\xi_{1t}\tilde{K}_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}\tilde{K}_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{1t} - \delta(U_t) \right\} \frac{\xi_{1t}}{Q_{1t-1}} \tag{79}$$

20. The capital return in sector 2,

$$R_{2kt} = \left\{ P_{mt}\alpha w^{\frac{1}{\sigma}} \frac{\Delta_t \tilde{Y}_t (\xi_{2t}\tilde{K}_{2t})^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}} (\xi_{1t}\tilde{K}_{1t})^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}} (\xi_{2t}\tilde{K}_{2t})^{\frac{\sigma-1}{\sigma}}]} + Q_{2t} - \delta(U_t) \right\} \frac{\xi_{2t}}{Q_{2t-1}} \tag{80}$$

21. Net capital created in sector 1,

$$\tilde{I}_{1nt} = \tilde{I}_{1t} - \delta(U_t)\xi_{1t}\tilde{K}_{1t} \tag{81}$$

22. Net capital created in sector 2,

$$\tilde{I}_{2nt} = \tilde{I}_{2t} - \delta(U_t)\xi_{2t}\tilde{K}_{2t} \tag{82}$$

23. Market value of an effective unit of capital in sector 1,

$$\begin{aligned} Q_{1t} =& 1 + f\left(\frac{\tilde{I}_{1nt}}{\tilde{I}_{1nt-1}} g_{At}\right) + \frac{\tilde{I}_{1nt}}{\tilde{I}_{1nt-1}} g_{At} f'\left(\frac{\tilde{I}_{1nt}}{\tilde{I}_{1nt-1}} g_{At}\right) \\ &- \mathbf{E}_t \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} g_{At+1} \left(\frac{\tilde{I}_{1nt+1}}{\tilde{I}_{1nt}}\right)^2 f'\left(\frac{\tilde{I}_{1nt}}{\tilde{I}_{1nt-1}} g_{At}\right) \end{aligned} \tag{83}$$

24. Market value of an effective unit of capital in sector 2,

$$\begin{aligned} Q_{2t} =& 1 + f\left(\frac{\tilde{I}_{2nt}}{\tilde{I}_{2nt-1}} g_{At}\right) + \frac{\tilde{I}_{2nt}}{\tilde{I}_{2nt-1}} g_{At} f'\left(\frac{\tilde{I}_{2nt}}{\tilde{I}_{2nt-1}} g_{At}\right) \\ &- \mathbf{E}_t \beta \frac{\tilde{\rho}_{t+1}}{\tilde{\rho}_t} g_{At+1} \left(\frac{\tilde{I}_{2nt+1}}{\tilde{I}_{2nt}}\right)^2 f'\left(\frac{\tilde{I}_{2nt}}{\tilde{I}_{2nt-1}} g_{At}\right) \end{aligned} \tag{84}$$

25. Pricing rule,

$$p_t^* = \frac{\varepsilon}{1-\varepsilon} \frac{\Gamma_t^a}{\Gamma_t^b} \tag{85}$$

26. Numerator in the pricing rule,

$$\Gamma_t^a = \tilde{\rho}_t P_{mt} \tilde{Y}_t + \beta\gamma \mathbf{E}_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\varepsilon} \Gamma_{t+1}^a \tag{86}$$

27. Denominator in the pricing rule,

$$\Gamma_t^b = \tilde{\rho}_t \tilde{Y}_t + \beta\gamma \mathbf{E}_t \left(\frac{\pi_{t+1}}{\pi}\right)^{\varepsilon-1} \Gamma_{t+1}^b \tag{87}$$

28. Evolution of inflation,

$$1 = \left[\gamma \left(\frac{\pi}{\pi_t}\right)^{1-\varepsilon} + (1-\gamma) p_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{88}$$

29. Price dispersion,

$$\Delta_t = (1-\gamma) p_t^{*-\varepsilon} + \gamma \left(\frac{\pi}{\pi_t}\right)^{-\varepsilon} \Delta_{t-1} \tag{89}$$

30. Resource constraint,

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_{1t} + \tilde{I}_{2t} + f\left(\frac{\tilde{I}_{1nt}}{\tilde{I}_{1nt-1}} g_{At}\right) \tilde{I}_{1nt} + f\left(\frac{\tilde{I}_{2nt}}{\tilde{I}_{2nt-1}} g_{At}\right) \tilde{I}_{2nt} \tag{90}$$

31. Evolution of capital in sector 1,

$$\tilde{K}_{1t+1} = (\xi_{1t} \tilde{K}_{1t} + \tilde{I}_{1nt}) \frac{1}{g_{At+1}} \tag{91}$$

32. Evolution of capital in sector 2,

$$\tilde{K}_{2t+1} = (\xi_{2t} \tilde{K}_{2t} + \tilde{I}_{2nt}) \frac{1}{g_{At+1}} \tag{92}$$

33. Monetary policy,

$$i_t = (1-\rho)\left[i + \kappa_\pi \pi_t + \kappa_y \left(\log \tilde{Y}_t - \log \tilde{Y}_{t-1} + \log g_{At}\right)\right] + \rho i_{t-1} \tag{93}$$

34. Fisher relation,

$$1 + i_t = R_{t+1} \mathbf{E}_t \pi_{t+1} \tag{94}$$

35. Assets share,

$$\frac{Q_{1t} S_{1t}}{Q_{2t} S_{2t}} = \frac{\kappa_t}{1-\kappa_t} \tag{95}$$

B Steady state

The transformed system presented in last section has a non-stochastic steady state. Assume that the function $\delta(\cdot)$ is such that the steady-state capacity utilization rate is equal to 1. By the steady-state version of (88), we obtain $p^* = 1$. It then follows from (89) that $\Delta = 1$. Combining (85), (86) and (87), we have $P_m = 1 - \frac{1}{\varepsilon}$, $\Gamma_a = P_m \Gamma_b = P_m \tilde{\rho} \tilde{Y}(1-\beta\gamma)$. Then we obtain a steady-state system of 29 equations for 29 variables: $\{\tilde{\rho}, \tilde{W}, L, \tilde{C}, R, \tilde{N}, \phi, \eta, \nu_1, \nu_2, \kappa, z, R_{1k}, R_{2k}, x_1, x_2, Q_1, Q_2, \tilde{S}_1, \tilde{S}_2, \tilde{K}_1, \tilde{K}_2, \tilde{Y}, \tilde{I}_{1n}, \tilde{I}_{2n}, \pi, i, \tilde{I}_1, \tilde{I}_2\}$.

1. Labor supply,

$$\tilde{\rho} \tilde{W} = \bar{\chi} L^{\varphi} \tag{96}$$

2. Marginal utility for consumption,

$$\tilde{\rho} = (\tilde{C} - h\tilde{C}/\bar{g}_A)^{-1} - \beta h (\tilde{C} \cdot \bar{g}_A - h\tilde{C})^{-1} \tag{97}$$

3. Risk-free rate,

$$\beta R = \bar{g}_A \tag{98}$$

4. Total intermediary demand for assets,

$$Q_1\tilde{S}_1 + Q_2\tilde{S}_2 = \phi\tilde{N} \tag{99}$$

5. Leverage ratio,

$$\phi = \frac{\eta}{\lambda - (\nu_1\kappa + \nu_2(1-\kappa))} \tag{100}$$

6. Gross growth rate of net worth,

$$z = [(R_{1k} - R)\kappa + (R_{2k} - R)(1-\kappa)]\phi + R \tag{101}$$

7. Gross growth rate in assets in sector 1,

$$x_1 = z \tag{102}$$

8. Gross growth rate in assets in sector 2,

$$x_2 = z \tag{103}$$

9. Expected discounted marginal gain to the banker of expanding assets in sector 1 by a unit, holding net worth constant,

$$\nu_1 = (1-\theta)\beta\frac{R_{1k} - R}{\bar{g}_A} + \beta\theta\frac{x_1\nu_1}{\bar{g}_A} \tag{104}$$

10. Expected discounted marginal gain to the banker of expanding assets in sector 1 by a unit, holding net worth constant,

$$\nu_2 = (1-\theta)\beta\frac{R_{2k} - R}{\bar{g}_A} + \beta\theta\frac{x_2\nu_2}{\bar{g}_A} \tag{105}$$

11. Expected discounted value of having another unit of net worth, holding assets constant,

$$\eta = (1-\theta) + \beta\theta\frac{z\eta}{\bar{g}_A} \tag{106}$$

12. Evolution of the banker's net worth,

$$\tilde{N} = \theta\left\{[(R_{1k} - R)\kappa + (R_{2k} - R)(1-\kappa)]\phi + R\right\}\frac{\tilde{N}}{\bar{g}_A} + \omega\left(Q_1\frac{\tilde{S}_1}{\bar{g}_A} + Q_2\frac{\tilde{S}_2}{\bar{g}_A}\right) \tag{107}$$

13. Risk control,

$$\kappa\sigma_1^2 + (1-\kappa)\sigma_2^2 \leq \bar{\sigma}^2 \tag{108}$$

14. Firm in sector 1 financing its capital acquisition,

$$\tilde{K}_1 = \frac{\tilde{S}_1}{\bar{g}_A} \tag{109}$$

15. Firm in sector 2 financing its capital acquisition,

$$\tilde{K}_2 = \frac{\tilde{S}_2}{\bar{g}_A} \tag{110}$$

16. Aggregate output,

$$\tilde{Y} = w^{\alpha-1}\left[w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\alpha\sigma}{\sigma-1}} L^{1-\alpha} \tag{111}$$

17. Utilization rate,

$$P_m\alpha\tilde{Y} = \delta'(1)(\bar{\xi}_1\tilde{K}_1 + \bar{\xi}_2\tilde{K}_2) \tag{112}$$

18. Labor demand,

$$P_m(1-\alpha)\frac{\tilde{Y}}{L} = \tilde{W} \tag{113}$$

19. The capital return in sector 1,

$$R_{1k} = \left\{ P_m \alpha w^{\frac{1}{\sigma}} \frac{\tilde{Y}(\bar{\xi}_1 \tilde{K}_1)^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}}(\bar{\xi}_1 \tilde{K}_1)^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2 \tilde{K}_2)^{\frac{\sigma-1}{\sigma}}]} + Q_1 - \delta(1) \right\} \frac{\bar{\xi}_1}{Q_1} \tag{114}$$

20. The capital return in sector 2,

$$R_{2k} = \left\{ P_m \alpha w^{\frac{1}{\sigma}} \frac{\tilde{Y}(\bar{\xi}_2 \tilde{K}_2)^{-\frac{1}{\sigma}}}{[w^{\frac{1}{\sigma}}(\bar{\xi}_1 \tilde{K}_1)^{\frac{\sigma-1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2 \tilde{K}_2)^{\frac{\sigma-1}{\sigma}}]} + Q_2 - \delta(1) \right\} \frac{\bar{\xi}_2}{Q_2} \tag{115}$$

21. Net capital created in sector 1,

$$\tilde{I}_{1n} = \tilde{I}_1 - \delta(1)\bar{\xi}_1 \tilde{K}_1 \tag{116}$$

22. Net capital created in sector 2,

$$\tilde{I}_{2n} = \tilde{I}_2 - \delta(1)\bar{\xi}_2 \tilde{K}_2 \tag{117}$$

23. Market value of an effective unit of capital in sector 1,

$$Q_1 = 1 + f\left(\bar{g}_A\right) + \bar{g}_A f'\left(\bar{g}_A\right) - \beta \bar{g}_A f'\left(\bar{g}_A\right) \tag{118}$$

24. Market value of an effective unit of capital in sector 2,

$$Q_2 = 1 + f\left(\bar{g}_A\right) + \bar{g}_A f'\left(\bar{g}_A\right) - \beta \bar{g}_A f'\left(\bar{g}_A\right) \tag{119}$$

25. Resource constraint,

$$\tilde{Y} = \tilde{C} + \tilde{I}_1 + \tilde{I}_2 + f\left(\bar{g}_A\right)\tilde{I}_{1n} + f\left(\bar{g}_A\right)\tilde{I}_{2n} \tag{120}$$

26. Evolution of capital in sector 1,

$$\tilde{K}_1 = (\bar{\xi}_1 \tilde{K}_1 + \tilde{I}_{1n})\frac{1}{\bar{g}_A} \tag{121}$$

27. Evolution of capital in sector 2,

$$\tilde{K}_2 = (\bar{\xi}_2 \tilde{K}_2 + \tilde{I}_{2n})\frac{1}{\bar{g}_A} \tag{122}$$

28. Monetary policy,

$$i = (1-\rho)\left[i + \kappa_\pi \pi + \kappa_y \cdot \log \bar{g}_A\right] + \rho \cdot i \tag{123}$$

29. Fisher relation,

$$1 + i = R \cdot \pi \tag{124}$$

30. Assets share,

$$\frac{Q_1 \tilde{S}_1}{Q_2 \tilde{S}_2} = \frac{\kappa}{1-\kappa} \tag{125}$$

C Linearized system

Let $\hat{x}_t = (x_t - x)/x$ denote the log deviation from steady state for any variable x_t. By standard linearization of New Keynesian model, we know the deviation of price dispersion Δ_t is of second-order. Thus we ignore the law of motion for the price dispersion. Moreover, the New Keynesian block can be summarized by the New-Keynesian Phillips curve. Hence, we can further eliminate $\hat{p}_t^*$, $\hat{\Gamma}_t^a$ and $\hat{\Gamma}_t^b$. Then, the log-linearized model can be summarized by a system of 31 equations for 31 variables: $\{\hat{\rho}_t, \hat{W}_t, \hat{L}_t, \hat{C}_t, \hat{R}_t, \hat{Q}_{1t}, \hat{S}_{1t}, \hat{Q}_{2t}, \hat{S}_{2t}, \hat{\phi}_t, \hat{N}_t, \hat{\eta}_t, \hat{\nu}_{1t}, \hat{\nu}_{2t}, \hat{\kappa}_t, \hat{z}_{t-1,t}, \hat{R}_{1kt}, \hat{R}_{2kt}, \hat{x}_{1t-1,t},$
$\hat{x}_{2t-1,t}, \hat{K}_{1t}, \hat{K}_{2t}, \hat{Y}_t, \hat{U}_t, \hat{P}_{mt}, \hat{I}_{1t}, \hat{I}_{2t}, \hat{I}_{1nt}, \hat{I}_{2nt}, \hat{\Pi}_t, \hat{i}_t\}$. And there are 6 exogenous variables driven by AR(1) processes: $\{\chi_t, g_{At}, \lambda_t, \bar{\sigma}_t^2, \xi_{1t}, \xi_{2t}\}$.

1. Labor supply,

$$\hat{\rho}_t + \hat{W}_t = \hat{\chi}_t + \varphi \hat{L}_t \tag{126}$$

2. Marginal utility for consumption,

$$\begin{aligned}\hat{\rho}_t = &\frac{\bar{g}_A}{\bar{g}_A - \beta h}[-\frac{\bar{g}_A}{\bar{g}_A - h}\hat{C}_t + \frac{h}{\bar{g}_A - h}(\hat{C}_{t-1} - \hat{g}_{At})] - \\ &\frac{\beta h}{\bar{g}_A - \beta h}\mathbf{E}_t[-\frac{\bar{g}_A}{\bar{g}_A - h}(\hat{C}_{t+1} + g_{At+1}) + \frac{h}{\bar{g}_A - h}\hat{C}_t]\end{aligned} \tag{127}$$

3. Risk-free rate,

$$\mathbf{E}_t(\hat{\rho}_{t+1} + \hat{R}_{t+1} - \hat{g}_{At+1}) = \hat{\rho}_t \tag{128}$$

4. Total intermediary demand for assets,

$$\hat{N}_t + \hat{\phi}_t = \kappa(\hat{Q}_{1t} + \hat{S}_{1t}) + (1-\kappa)(\hat{Q}_{2t} + \hat{S}_{2t}) \tag{129}$$

5. Leverage ratio,

$$(\lambda - \nu)(\hat{\eta}_t - \hat{\phi}_t) = \lambda\hat{\lambda}_t - \kappa\nu_1\hat{\nu}_{1t} - (1-\kappa)\nu_2\hat{\nu}_{2t} + (\nu_2 - \nu_1)\kappa\hat{\kappa}_t \tag{130}$$

6. Gross growth rate of net worth,

$$\begin{aligned}((R_k - R)\phi + R)\hat{z}_{t-1,t} = &\kappa\phi R_{1k}\hat{R}_{1kt} + (1-\kappa)\phi R_{2k}\hat{R}_{2kt} + (1-\phi)R\hat{R}_t \\ &+ (R_{1k} - R_{2k})\phi\kappa\hat{\kappa}_{t-1} + (R_k - R)\phi\hat{\phi}_{t-1}\end{aligned} \tag{131}$$

7. Gross growth rate in assets in sector 1,

$$\hat{x}_{1t-1,t} + \hat{\kappa}_{t-1} + \hat{\phi}_{t-1} = \hat{\kappa}_t + \hat{\phi}_t + \hat{z}_{t-1,t} \tag{132}$$

8. Gross growth rate in assets in sector 2,

$$\hat{x}_{2t-1,t} - \frac{\kappa}{1-\kappa}\hat{\kappa}_{t-1} + \hat{\phi}_{t-1} = -\frac{\kappa}{1-\kappa}\hat{\kappa}_t + \hat{\phi}_t + \hat{z}_{t-1,t} \tag{133}$$

9. Expected discounted marginal gain to the banker of expanding assets in sector 1 by a unit, holding net worth constant

$$\begin{aligned}\hat{\nu}_{1t} = \mathbf{E}_t\{&\hat{\rho}_{t+1} - \hat{\rho}_t - \hat{g}_{At+1} + \frac{(\bar{g}_A - \beta\theta x)R_{1k}}{\bar{g}_A(R_{1k} - R)}\hat{R}_{1kt+1} \\ &- \frac{(\bar{g}_A - \beta\theta x)R}{\bar{g}_A(R_{1k} - R)}\hat{R}_{t+1} + \frac{\beta\theta x}{\bar{g}_A}(\hat{\nu}_{1t+1} + \hat{x}_{1tt+1})\}\end{aligned} \tag{134}$$

10. Expected discounted marginal gain to the banker of expanding assets in sector 2 by a unit, holding net worth constant

$$\begin{aligned}\hat{\nu}_{2t} = \mathbf{E}_t\{&\hat{\rho}_{t+1} - \hat{\rho}_t - \hat{g}_{At+1} + \frac{(\bar{g}_A - \beta\theta x)R_{2k}}{\bar{g}_A(R_{2k} - R)}\hat{R}_{2kt+1} \\ &- \frac{(\bar{g}_A - \beta\theta x)R}{\bar{g}_A(R_{2k} - R)}\hat{R}_{t+1} + \frac{\beta\theta x}{\bar{g}_A}(\hat{\nu}_{2t+1} + \hat{x}_{2tt+1})\}\end{aligned} \tag{135}$$

11. Expected discounted value of having another unit of net worth, holding assets constant,

$$\hat{\eta}_t = \frac{\beta\theta z}{\bar{g}_A}\mathbf{E}_t(\hat{\rho}_{t+1} - \hat{\rho}_t + \hat{\eta}_{t+1} + \hat{z}_{t,t+1} - \hat{g}_{At+1}) \tag{136}$$

12. Evolution of the banker's net worth,

$$\bar{g}_A(\hat{N}_t + \hat{g}_{At}) = \theta\phi R_k(\frac{\kappa R_{1k}}{R_k}\hat{R}_{1kt} + \frac{(R_{1k} - R_{2k})\kappa}{R_k}\hat{\kappa}_t + \frac{(1-\kappa)R_{2k}}{R_k}\hat{R}_{2kt})$$
$$+ \theta(1-\phi)R\hat{R}_t + \theta\phi(R_k - R)\hat{\phi}_{t-1} + \theta((R_k - R)\phi + R)\hat{N}_{t-1}$$
$$+ (\bar{g}_A - \theta((R_k - R)\phi + R))(\kappa(\hat{Q}_{1t} - \hat{S}_{1t}) + (1-\kappa)(\hat{Q}_{2t} - \hat{S}_{2t})) \tag{137}$$

13. Risk control,

$$\kappa(\sigma_1^2 - \sigma_2^2)\hat{\kappa}_t = 2\sqrt{\kappa\sigma_1^2 + (1-\kappa)\sigma_2^2}\hat{\bar{\sigma}}_t \tag{138}$$

14. Relative size of capital value,

$$\hat{Q}_{2t} + \hat{S}_{2t} = \hat{Q}_{1t} + \hat{S}_{1t} - \frac{1}{1-\kappa}\hat{\kappa}_t \tag{139}$$

15. Firm financing its capital acquisition in sector 1,

$$\hat{K}_{1t+1} = \hat{S}_{1t} - \hat{g}_{At+1} \tag{140}$$

16. Firm financing its capital acquisition in sector 2,

$$\hat{K}_{2t+1} = \hat{S}_{2t} - \hat{g}_{At+1} \tag{141}$$

17. Aggregate output,

$$\hat{Y}_t = \alpha\hat{U}_t + \alpha\frac{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}}(\hat{\xi}_{1t} + \hat{K}_{1t}) + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}(\hat{\xi}_{2t} + \hat{K}_{2t})}{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}} + (1-\alpha)\hat{L}_t \tag{142}$$

18. Utilization rate,

$$\hat{P}_{mt} + \hat{Y}_t - \hat{U}_t = \frac{\delta''(1)}{\delta'(1)}\hat{U}_t + \frac{\bar{\xi}_1\tilde{K}_1}{\bar{\xi}_1\tilde{K}_1 + \bar{\xi}_2\tilde{K}_2}(\hat{\xi}_{1t} + \hat{K}_{1t}) + \frac{\bar{\xi}_2\tilde{K}_2}{\bar{\xi}_1\tilde{K}_1 + \bar{\xi}_2\tilde{K}_2}(\hat{\xi}_{2t} + \hat{K}_{2t}) \tag{143}$$

19. Labor demand,

$$\hat{P}_{mt} + \hat{Y}_t - \hat{L}_t = \hat{W}_t \tag{144}$$

20. The capital return in sector 1,

$$\frac{Q_1 R_{1k}}{\bar{\xi}_1}(\hat{Q}_{1t-1} + \hat{R}_{1kt} - \hat{\xi}_{1t}) = (\frac{Q_1 R_{1k}}{\bar{\xi}_1} - Q_1 + \delta(1))(\hat{P}_{mt} + \hat{Y}_t - \frac{1}{\sigma}(\hat{\xi}_{1t} + \hat{K}_{1t})$$
$$-\frac{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}}(\hat{\xi}_{1t} + \hat{K}_{1t}) + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}(\hat{\xi}_{2t} + \hat{K}_{2t})}{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}}\frac{\sigma-1}{\sigma}) + Q_1\cdot\hat{Q}_{1t} - \delta'(1)\hat{U}_t \tag{145}$$

21. The capital return in sector 2,

$$\frac{Q_2 R_{2k}}{\bar{\xi}_2}(\hat{Q}_{2t-1} + \hat{R}_{2kt} - \hat{\xi}_{2t}) = (\frac{Q_2 R_{2k}}{\bar{\xi}_2} - Q_2 + \delta(1))(\hat{P}_{mt} + \hat{Y}_t - \frac{1}{\sigma}(\hat{\xi}_{2t} + \hat{K}_{2t})$$
$$-\frac{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}}(\hat{\xi}_{1t} + \hat{K}_{1t}) + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}(\hat{\xi}_{2t} + \hat{K}_{2t})}{w^{\frac{1}{\sigma}}(\bar{\xi}_1\tilde{K}_1)^{1-\frac{1}{\sigma}} + (1-w)^{\frac{1}{\sigma}}(\bar{\xi}_2\tilde{K}_2)^{1-\frac{1}{\sigma}}}\frac{\sigma-1}{\sigma}) + Q_2\cdot\hat{Q}_{2t} - \delta'(1)\hat{U}_t \tag{146}$$

22. Market value of an effective unit of capital in sector 1,

$$(1 + f(\bar{g}_A) + (1-\beta)\bar{g}_A f'(\bar{g}_A))\hat{Q}_{1t} = (2\bar{g}_A f'(\bar{g}_A) + \bar{g}_A^2 f''(\bar{g}_A))(\hat{I}_{1nt} + \hat{g}_{At} - \hat{I}_{1nt-1}) -$$
$$\mathbf{E}_t\beta[\bar{g}_A f'(\bar{g}_A)(\hat{\rho}_{t+1} - \hat{\rho}_t + \hat{g}_{At+1} + 2\hat{I}_{1nt+1} - 2\hat{I}_{1nt}) + \bar{g}_A^2 f''(\bar{g}_A)(\hat{I}_{1nt} - \hat{I}_{1nt-1} + \hat{g}_{At})] \tag{147}$$

23. Market value of an effective unit of capital in sector 2,

$$(1+f(\bar{g}_A)+(1-\beta)\bar{g}_A f'(\bar{g}_A))\hat{Q}_{2t} = (2\bar{g}_A f'(\bar{g}_A)+\bar{g}_A^2 f''(\bar{g}_A))(\hat{I}_{2nt}+\hat{g}_{At}-\hat{I}_{2nt-1})-$$
$$\mathbf{E}_t\beta[\bar{g}_A f'(\bar{g}_A)(\hat{\rho}_{t+1}-\hat{\rho}_t+\hat{g}_{At+1}+2\hat{I}_{2nt+1}-2\hat{I}_{2nt})+\bar{g}_A^2 f''(\bar{g}_A)(\hat{I}_{2nt}-\hat{I}_{2nt-1}+\hat{g}_{At})] \tag{148}$$

24. New-Keynesian Phillips curve,

$$\hat{\Pi}_t = \beta\mathbf{E}_t\hat{\Pi}_{t+1}+(1-\gamma)(1-\beta\gamma)\gamma\hat{P}_{mt} \tag{149}$$

25. Resource constraint,

$$\hat{Y}_t = \frac{\tilde{C}}{\tilde{Y}}\hat{C}_t+\frac{\tilde{I}_1}{\tilde{Y}}\hat{I}_{1t}+\frac{\tilde{I}_2}{\tilde{Y}}\hat{I}_{2t}+\frac{\tilde{I}_{1n}}{\tilde{Y}}(f'(\bar{g}_A)\bar{g}_A+f(\bar{g}_A))\hat{I}_{1nt}-\frac{\tilde{I}_{1n}}{\tilde{Y}}f'(\bar{g}_A)\bar{g}_A(\hat{I}_{1nt-1}-\hat{g}_{At})+$$
$$\frac{\tilde{I}_{2n}}{\tilde{Y}}(f'(\bar{g}_A)\bar{g}_A+f(\bar{g}_A))\hat{I}_{2nt}-\frac{\tilde{I}_{2n}}{\tilde{Y}}f'(\bar{g}_A)\bar{g}_A(\hat{I}_{2nt-1}-\hat{g}_{At}) \tag{150}$$

26. Net capital created in sector 1,

$$\frac{\tilde{I}_1}{\tilde{Y}}\hat{I}_{1t} = \frac{\tilde{I}_{1n}}{\tilde{Y}}\hat{I}_{1nt}+\bar{\xi}_1\frac{\tilde{K}_1}{\tilde{Y}}[\delta'(1)\hat{U}_t+\delta(1)(\hat{K}_{1t}+\hat{\xi}_{1t})] \tag{151}$$

27. Net capital created in sector 2,

$$\frac{\tilde{I}_2}{\tilde{Y}}\hat{I}_{2t} = \frac{\tilde{I}_{2n}}{\tilde{Y}}\hat{I}_{2nt}+\bar{\xi}_2\frac{\tilde{K}_2}{\tilde{Y}}[\delta'(1)\hat{U}_t+\delta(1)(\hat{K}_{2t}+\hat{\xi}_{2t})] \tag{152}$$

28. Evolution of capital in sector 1,

$$\hat{K}_{1t+1}+\hat{g}_{At+1} = \frac{\bar{\xi}_1}{\bar{\xi}_1+\tilde{I}_{1n}/\tilde{K}_1}(\hat{\xi}_{1t}+\hat{K}_{1t})+\frac{\tilde{I}_{1n}/\tilde{K}_1}{\bar{\xi}_1+\tilde{I}_{1n}/\tilde{K}_1}\hat{I}_{1nt} \tag{153}$$

29. Evolution of capital in sector 2,

$$\hat{K}_{2t+1}+\hat{g}_{At+1} = \frac{\bar{\xi}_2}{\bar{\xi}_2+\tilde{I}_{2n}/\tilde{K}_2}(\hat{\xi}_{2t}+\hat{K}_{2t})+\frac{\tilde{I}_{2n}/\tilde{K}_2}{\bar{\xi}_2+\tilde{I}_{2n}/\tilde{K}_2}\hat{I}_{2nt} \tag{154}$$

30. Monetary policy,

$$i\cdot\hat{i}_t = (1-\rho)\left[\kappa_\pi\Pi\cdot\hat{\Pi}_t+\kappa_y\left(\hat{Y}_t-\hat{Y}_{t-1}+\hat{g}_{At}\right)\right]+\rho\cdot i\cdot\hat{i}_{t-1} \tag{155}$$

31. Fisher relation,

$$\frac{i}{1+i}\hat{i}_t = \hat{R}_{t+1}+\mathbf{E}_t\hat{\Pi}_{t+1} \tag{156}$$

The log-linearized shock processes are listed below.

1. The TFP shock,

$$\hat{g}_{At} = \rho_A\hat{g}_{At-1}+\varepsilon_{At} \tag{157}$$

2. The labor supply shock,

$$\hat{\chi}_t = \rho_\chi\hat{\chi}_{t-1}+\varepsilon_{\chi t} \tag{158}$$

3. The capital quality shock in sector 1,

$$\hat{\xi}_{1t} = \rho_{\xi_1}\hat{\xi}_{1t-1}+\varepsilon_{\xi_1 t} \tag{159}$$

4. The capital quality shock in sector 2,

$$\hat{\xi}_{2t} = \rho_{\xi_2}\hat{\xi}_{2t-1}+\varepsilon_{\xi_2 t} \tag{160}$$

5. The financial shock,

$$\hat{\lambda}_t = \rho_\lambda\hat{\lambda}_{t-1}+\varepsilon_{\lambda t} \tag{161}$$

6. The financial regulation shock,

$$\hat{\bar{\sigma}}_t = \rho_{\bar{\sigma}}\hat{\bar{\sigma}}_{t-1}+\varepsilon_{\bar{\sigma}t} \tag{162}$$